Long Memory Volatility and Bernoulli Jumps in Daily Crypto Currency Prices*

일별 암호화폐 가격들에서 나타나는 장기기억 변동성과 베르누이 점프현상

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This paper investigates the intrinsic time series properties of daily crypto currency prices, the long memory volatility and the jumps. For the purpose, this paper first adopts the simple FIGARCH model to analyze the long memory volatility process of the crypto currency prices and finds that there exists the long memory volatility in the daily returns. But, the jumps are found to be significant in the daily returns so that the simple FIGARCH model appears to be inadequate. Thus, this paper uses a normal mixture distribution which includes the Bernoulli jumps in the daily returns. In particular, the jumps appear to make the long memory volatility more significant in the daily returns. The results imply that using simple FIGARCH model without the jumps may yield the incorrect long memory volatility process of the crypto currencies and result in ineffective risk management and portfolio optimization in the markets. Thus, the FIGARCH model with allowing for the jump process could be more appropriate in the aspects of risk management and investment purpose forecasting the risk in such an investment as this market attracts increasing attractions from regulators and investors.

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I. Introduction

Since the inception of the crypto currencies including the Bitcoin, they have gained growing attention from media, academics and finance industry and in recent years the global interest in Bitcoin and the crypto currencies has spiked dramatically. Catania and Grassi (2017) presents that the reasons for the great interests in the crypto currencies are that some central banks are exploring the use of the crypto currencies and a big number of companies and banks created the Enterprise Ethereum Alliance to use the crypto currencies and the related technology called "Block chain". All this interest has been reflected on the crypto currency market capitalization exploding from 19 billions in February 2017 to 142 billions in August 2017. ElBahrawy *et al.* (2017) provides the comprehensive analysis of the crypto currencies considering various issues such as market shares and turnovers. Hence, it is time to investigate in deep this new market that is becoming an important piece of financial technology.

The crypto currencies including Bitcoin, Ethereum and Ripple are a kind of digital decentralized currency in the sense that it is not created by any central authority and may be immune to any central bank's interferences and that it uses cryptography to regulate the creation and transactions of the exchange unit. The crypto currency market is the retail focused and highly speculative market that leaks a legal and regulatory framework compared to other financial asset markets like stock markets and FX markets (Shaw, 2017). At present, it is estimated that the transaction volume in the crypto currency market exceeds 100 million USD per day, and the number of hedge funds that trade the crypto currencies has recently reached almost 100 for the first time, of which more than three-quarters were launched in 2017.

Even though there has recently been the exponential growth of the crypto

currency market, the market is rather young and therefore still mostly unexplored in many issues. The key issue to be analyzed is the dynamic price behavior of the crypto currencies in order to gauge the risks related to investment in crypto currencies (Caporale *et al.*, 2017). As pointed out by Baek and Elbeck (2015), the crypto currencies including Bitcoin are mostly used for speculative purposes causing extreme volatility and bubbles so that they have experienced numerous episodes of extreme volatility and apparent discontinuities in the price process. Dwyer (2014) and Carrick (2016) present that the crypto currency market is more volatile than other financial markets.

One reason is that the absence of solid history and exhaustive legal framework make the crypto currencies very speculative investments. Since they do not rely on the stabilizing policy of a central bank, their reactions to new information, whether fundamental or speculative, result in high and persistent volatilities relative to traditional currencies. Another reason is that the relative illiquidity of the crypto currency market with no official market maker makes it fundamentally fragile to large trading volumes and to market imperfections, and thus more prone to large changes or jumps than other types of traded financial assets such as stocks, commodities and foreign exchange rates (Scaillet *et al.*, 2017). Thus, the persistent volatility and the jumps appear to be key features of the price process of the crypto currencies.

The specification of the jumps and the persistent volatility in the crypto currencies appear to be quite complex because they show extreme observations, asymmetries and nonlinear characteristics. In particular, the papers of Bouri *et al.* (2016), Bariviera *et al.* (2017) and Caporale *et al.* (2017) find the persistent long memory property in Bitcoin and other crypto currencies. But, the literature on the persistent long memory volatility modelling of the crypto currency is very rare[1] even though there is a wide

literature on econometric techniques which have used the analysis of the long memory volatility in many financial data. Also, some papers like Gronwald (2015) and Scaillet *et al.* (2017) present strong evidence of the jumps behavior and apparent discontinuities in the price process of the crypto currencies including Bitcoin together with the extreme volatility. Scaillet *et al.* (2017) analyzes the Bitcoin prices by using high frequency transaction level data and assert that the jumps are the essential component of the dynamics of the Bitcoin prices and the jumps cluster in time. They explain that order flow imbalance, illiquidity and the dominant effect of aggressive traders are significant factors driving the occurrence of the jumps.

The main concern of this paper is to analyze the dynamic price behavior of crypto currencies focusing on the jumps and the long memory volatility by using the daily prices of the three main crypto currencies, Bitcoin, Ethereum and Ripple. Thus, this paper first uses the FIGARCH model with the normality assumption to examine the behavior of long memory volatility process since the FIGARCH model has been considered to be quite useful to represent the autocorrelations of the squared returns of most financial asset prices, which decay at very slow hyperbolic rate (Baillie *et al.*, 1996). In particular, the attraction of the FIGARCH model is that it is very changeable to represent middle ranges of persistence. However, the simple FIGARCH model with the assumption of a symmetric normal distribution is unlikely to represent the asymmetric jump process of the daily returns of the crypto currencies because the long memory volatility cannot be specified exactly without the jump specification.

For the reason, it seems to be very appropriate to use the jump diffusion

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1) Chou *et al.* (2017) and Shaw (2017) provided the GARCH model in order to investigate the volatility process of crypto currencies.

model of Press (1967) to represent the daily returns of the three crypto currencies and properly account for the jumps. Since the jumps and the long memory volatility contain different statistical and economic motivations, this paper invokes the FIGARCH model with Bernoulli jump process that consider the two features simultaneously. Thus, this paper shows the combined model performs quite well and improves the estimates of the long memory volatility parameters because the misspecification without considering the jumps may provide distorted the long memory volatility parameters.

There are two contributions of this paper. First, this paper can provide the more appropriate specification model for the volatility process of the crypto currencies by examining the behavior of long memory volatility using the FIGARCH model with the normality assumption. And, the second contribution of this paper is to detect the presence of the jumps in the crypto currency market and explain the dynamics of the jumps with the long memory volatility. Thus, the jumps have significant impacts on the crypto currencies and induce a persistent volatility in the prices. The results imply that using simple FIGARCH model may yield incorrect long memory volatility process of the crypto currencies without considering the jumps and result in ineffective risk management and portfolio optimization in the markets. Since modeling volatility is crucial for risk management, the results of this paper can be very useful to investors when accounting for future volatility and implementing hedging strategies. Thus, this paper provides important implications in the aspects of risk management and investment purpose for choosing a reliable model forecasting the risk in such an investment as this market attracts increasing attraction from regulators and investors.

The organization of this paper follows: Section II describes the key aspects and the basic descriptive statistics for the daily prices of the three major

crypto currencies, Bitcoin, Ethereum and Ripple in terms of the USD. Section III presents the basic analysis of the daily returns series of the crypto currency prices. The FIGARCH model of Baillie *et al.* (1996) is discussed for the analysis of the daily returns. Section IV then describes the FIGARCH model combined with Bernoulli jump process to represent the jumps, the high excess kurtosis and the long memory volatility in the daily returns. Finally, section V provides a conclusion briefly.

II. Basic Descriptive Statistics of Daily Crypto Currency Prices

Even though hundreds of other crypto currencies have been created since the Bitcoin was introduced in 2009, some of them do not represent serious attempts at establishing a foothold in the market and others are quite young with just a few useful observations due to quite low trading volume before 2017. However, since 2017 many central banks have explored the use of the crypto currencies and a big number of companies and banks have created the Enterprise Ethereum Alliance to use of the crypto currencies by the related technology of blockchain so that the trading volumes and activities in the crypto currency markets became increased quite significantly (Catania and Grassi, 2017). In particular, the market capitalizations increased from approximately 18 billion US Dollar at the beginning of 2017 to nearly 600 billion at the end of the year and the high returns have attracted more investors in the markets (Caporale and Zekokh, 2019). Also two big exchangers, the Chicago Mercantile Exchanges (CME) and the Chicago Board Options Exchanges (CBOE) started to trade futures on Bitcoin in 2017

(Caporale and Zekokh, 2019).

For the reason this paper uses the three major crypto currencies, Bitcoin, Ethereum and Ripple, which have been the highest market capitalizations and traded in the markets most actively with huge booms and busts since 2017. Since the crypto currency market is open 24 hours in a day and seven days in a week, the crypto currency data used in this paper is the daily closing price which is the latest recorded price in UTC (Coordinated Universal Time) of three major crypto currencies (Bitcoin, Ethereum, Ripple) in terms of US Dollar (USD). The daily data of the three major crypto currencies are obtained from CoinMarketCap (https://coinmarketcap.com/)[2] and sampled from January 1, 2017 to May 31, 2019 with 881 observations.

This section describes some specific details of the daily crypto currencies (Bitcoin, Ethereum and Ripple) and provides basic descriptive statistics for the daily prices of the crypto currencies. In order to analyze the volatility behaviors of the crypto currencies, this paper calculates the returns data by using the daily closing price of the crypto currencies. The daily return at day(t) is defined as;

$$y_t = 100*[\ln(p_t) - \ln(p_{t-1})] \tag{1}$$

where p_t is the daily closing price of the crypto currency

Figures 1(a) through(c) show the movements of the daily returns of the three crypto currencies. All the daily returns of the three crypto currencies are very closed to zero but there exist significant volatility clustering over the entire sample periods. And, the excessive turbulences in the market seem to induce significant jumps and heavy tailed, undefined variances of unconditional returns

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2) The data sets provided by the CoinMarketCap are also used in some previous studies including Caporale *et al.* (2017), Caporale and Plastun (2017) and Catania and Grassi (2017) so that the credibility of the data is proved by the previous studies.
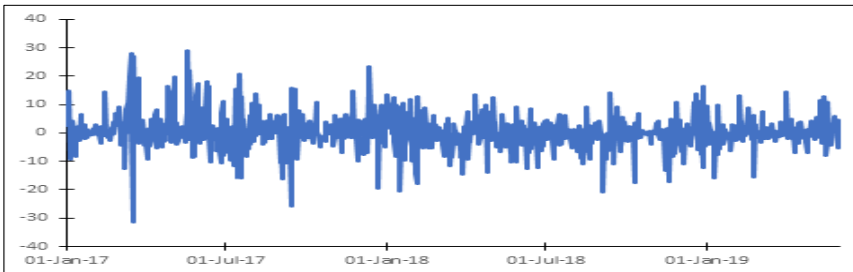
phenomenon, as in the other financial assets studied by Koedijk *et al.* (1990).

〈Figure 1〉 Daily Returns of the Three Crypto Currencies
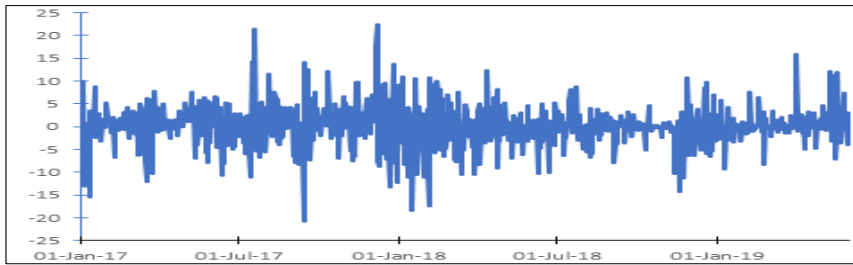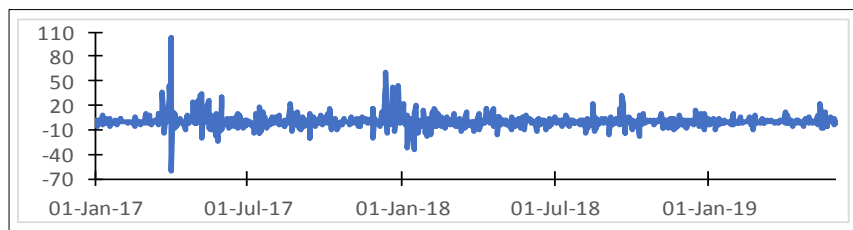
(a) Bitcoin

(b) Ethereum

(c) Ripple

The details of the basic statistics for the daily returns of the three crypto currencies are provided in Table 1. The means of the daily returns are found to be 0.24, 0.39 and 0.48 for Bitcoin, Ethereum and Ripple respectively, which are close to zero and indistinguishable at the standard significance level. It is also noted that the daily returns of the Ripple have the largest maximum and

the those of the Ethereum have the lowest minimum. And, all the daily returns are not normally distributed since the skewness (m3) are 0.12, 0.48 and 2.85 and the kurtosis (m4) are 7.37, 6.73 and 34.36 for Bitcoin, Ethereum and Ripple, which are quite different from the values of the normal distribution and the values are statistically significant.[3]

In particular, the high level of the excess kurtosis statistics implies the fat-tailed distributions of for the daily returns of the three crypto currencies and may be closely related to the jumps caused in the crypto currency market due to several factors. For example, one factor is that the absence of solid history and exhaustive legal framework make the crypto currencies very speculative investments and react sensitively to new information, whether fundamental or speculative. And, the other factor is the relative illiquidity of the crypto currency market with no official market maker makes it fundamentally fragile to market imperfections (Scaillet *et al.*, 2017). Thus, the crypto currency market may be prone to experience the jumps in the price process.

The Ljung-Box test statistics, Q(20) which is for the test of the serial correlations generally can not accept the hypothesis that there exit serial correlations in the level returns of the daily returns at the standard significance levels so that the daily returns do not include any serial correlations in the mean process. But, another test statistics, $Q^2(20)$ from the squared returns are 130.52, 123.91 and 156.22 for Bitcoin, Ethereum and Ripple respectively, which are also statistically significant, and they indicate that there exist very persistent autocorrelations in the volatility process with the strong ARCH effects. The serial correlation appears to be the most significant in the daily returns of the Ripple prices.

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3) Jarque and Bera (1987) shows that the standard errors of the sample skewness and the sample kurtosis in normal distributions are $(6/T)^{1/2}$ and $(24/T)^{1/2}$.

〈Table 1〉 Descriptive Statistics of the Daily Returns of the Crypto Currencies

|	Bitcoin	Ethereum	Ripple
Maximum	22.519	29.013	102.736
Minimum	-24.753	-31.547	-61.627
Mean	0.244	0.397	0.481
Variance	19.581	37.523	74.690
m3	0.128	0.484	2.851
m4	7.378	6.730	34.369
Q(20)	31.057	31.705	51.900
$Q^2(20)$	130.521	123.910	156.226

Note: The Ljung-Box test statistics, Q(20) and $Q^2(20)$ are calculated at 20 degrees of freedom for serial correlation in the daily returns and squared daily returns. m3 and m4 are the values of the sample skewness and kurtosis.

Figures 2(a) through (c) present the autocorrelation function of the returns, squared returns and absolute returns of the daily prices of the crypto currencies. The autocorrelations in the returns of all the crypto currencies appear to be small and insignificant at standard levels. On the other hand, the autocorrelations of the squared returns and the absolute returns of the crypto currencies are found to decay quite slowly and persistently representing the typical long memory volatility feature as presented in Baillie (1996).
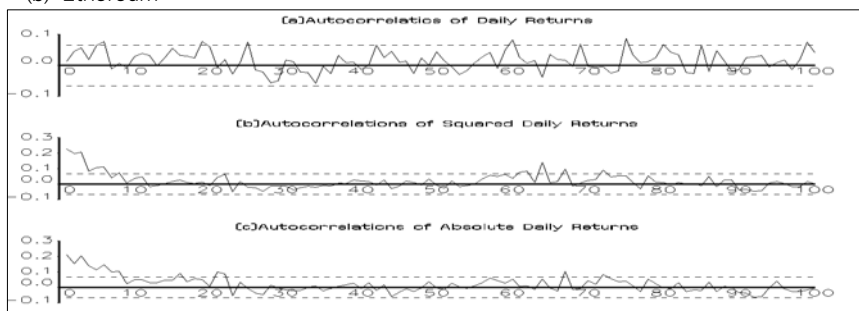
The long memory volatility features of the daily returns of the crypto currencies are found to be quite apparent in the autocorrelations of the squared and absolute returns and are the most significant in the autocorrelation functions of the absolute returns as in many financial time series assets (Granger and Ding, 1996). And, the returns of the Ripple prices show that the degree of the long memory volatility is the most significant. These findings of the correlograms appear to be quite consistent with the basic descriptive statistics in Table 1 for the daily returns of the three crypto currencies.

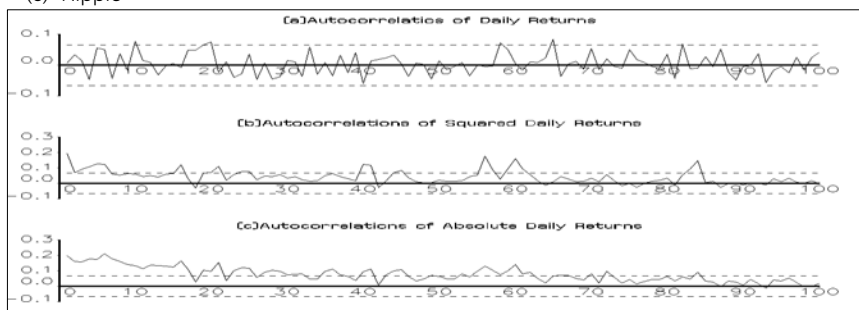〈Figure 2〉 Correlograms of Daily returns of three crypto currencies

(a) Bitcoin
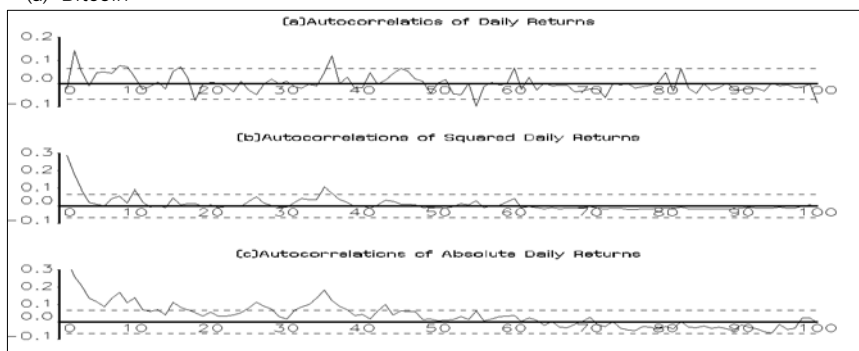
(b) Ethereum

(c) Ripple

III. Long Memory Volatility in Daily Crypto Currency Prices

This section describes the long memory volatility of the daily returns series of the three crypto currencies. The long memory volatility process has become the well documented feature of many financial assets such as stock prices and exchange rates (Baillie, 1996). Many long memory volatility models have been used to specify the long memory volatility process of the financial time series data. Among them, some methodologies are to incorporate this feature in dynamic models; fractional integration model (Baillie et al, 1996), stochastic volatility model (Breidt *et al.*, 1998), multi components model (Andersen *et al.*, 2006), and heterogeneous autoregressive model (Corsi, 2009).

However, not many studies have focused on the long memory volatility in the crypto currency market. Some exceptions are the papers of Bouri *et al.* (2016) and Catania and Grassi (2017) which find the long memory property in the crypto currency prices. While Bouri *et al.* (2016) detects the long memory volatility from the Bitcoin prices, Catania and Grassi (2017) and Caporale *et al.* (2017) find statistical evidence of the long memory volatility process in the four different crypto currencies (Bitcoin, Ethereum, Ripple and Litecoin) and argue that the long memory property has a substantial contribution in the volatility dynamics of the crypto currency prices.

This paper also investigates the characteristics of the long memory volatility process in the daily returns of the three crypto currencies (Bitcoin, Ethereum and Ripple) by applying the FIGARCH process of Baillie *et al.* (1996). The following model consistent with the basic stylized properties presented in Section II is the ARMA(*m,n*)-FIGARCH(*p, d, q*) model,

$$y_t = \mu + \psi(L)y_{t-1} + \theta(L)\epsilon_t \qquad (2)$$

$$\epsilon_t = z_t \sigma_t \tag{3}$$

$$[1 - \beta(L)]\sigma_t^2 = \omega + [1 - \beta(L) - \phi(L)(1 - L)^d]\epsilon_t^2 \tag{4}$$

where y_t is the daily returns, z_t follows i.i.d.N(0,1), μ and ω are constants, ψ (L), θ(L), β(L) and ϕ(L) are polynomials in the lag operator, and d is the long memory parameter.

The FIGARCH process is quite general since it can represent the autocorrelations of squared returns which decay at very slow hyperbolic rate as presented in Baillie (1996). When d = 0 and p = q = 1, equation (4) represents the standard GARCH (1,1) model; and when d = p = q = 1, equation (4) can be the Integrated GARCH, or IGARCH (1,1) model showing the complete persistence of the conditional variance to a shock in squared returns. The FIGARCH process shows the impulse response weights, $\sigma_t^2 = \omega/(1 - \beta) + \lambda(L)\varepsilon_t^2$, where for large lag k, $\lambda_k \approx k^{d-1}$, which represent the long memory property or "Hurst effect" of the very slow and hyperbolic decay. The hyperbolic decay is consistent with $\rho \approx c_1 k^{2d-1}$ as k gets large, where c_1 is a scalar and d is the long memory parameter. This kind of persistence appears to be very slow and such slow and hyperbolic decay is also known as the "Hurst phenomenon." The Hurst coefficient is, $H = d + 0.5$. If $d = 1$ and $H = 1.5$, the autocorrelation function does not decay so that the series shows a unit root property. If $d = 0$ and $H = 0.5$, the autocorrelation function decays exponentially and the series shows a stationary property. But for $0 < d < 1$ and $0.5 < H < 1.5$, the series presents quite slow and hyperbolic rate of decay in the autocorrelations. The strong advantage of the FIGARCH process is that for $0 < d < 1$, it is very changeable enough to represent middle ranges of persistence.

The equations (2) through (4) are estimated based on the non-linear optimization procedures by maximizing the Gaussian log likelihood function:

$$\ln(L;\Theta) = -\left(\frac{T}{2}\right)\ln(2\pi) - \left(\frac{1}{2}\right)\sum_{t=1}^{n}[\ln(\sigma_t^2) + \epsilon_t^2\sigma_t^{-2}] \tag{5}$$

where Θ is a vector of the unknown parameters.

But, it has been presented that most asset returns may not be represented well by the assumption of normal distribution of z_t in equation (3). Thus, the inference is calculated by using the QMLE of Bollerslev and Wooldridge (1992), which is still valid even though z_t is not Gaussian. It should be noted that the vector of parameter estimates on the ARMA-FIGARCH model from maximizing equation (5) using a sample of T observations with non-normal z_t by $\widehat{\Theta}_T$, the limiting distribution of $\widehat{\Theta}_T$ is

$$T^{\frac{1}{2}}(\widehat{\Theta}_T - \Theta_0) \rightarrow N[0, A(\Theta_0)^{-1}B(\Theta_0)A(\Theta_0)^{-1}] \tag{6}$$

where A(.) and B(.) represent the Hessian and outer product gradient respectively, and denotes the vector of true parameter values.

Equation (6) is applied to find the values of the robust standard errors in the subsequent results of this paper by evaluating the Hessian and outer product gradient matrices at the point for practical implementation.

The most appropriate ARMA and FIGARCH model is selected for the autocorrelation structure of the daily returns data. In particular, this paper uses the standard portmanteau test statistic,

$$Q(m) = T(T+2)\sum_{j=1}^{m} r_j^2/(T-j) \tag{7}$$

where r_j is the j'th order sample autocorrelation from the residuals.

The portmanteau test statistic uses similar degrees of freedom adjustments in the squared standardized residuals when they test for omitted conditional heteroscedasticity and ARCH effects. This adjustment is based on the suggestions by Diebold (1988). And, the values of the sample skewness and kurtosis calculated from the standardized residuals (m_3 and m_4) are also taken accounted. The best parametric specification of the model, which can represent the degree of autocorrelation in the mean process and variance process of the daily returns are found to be the Martingale-FIGARCH(1, d, 1) model for the daily returns of the three crypto currencies.

Table 2 shows the results of the FIGARCH model estimation for the daily returns of the three crypto currencies, which represents the long memory volatility property. The estimation result of the FIGARCH model shows that the long memory volatility parameters (d) are found to be 0.37, 0.26 and 0.54 for the daily returns of the Bitcoin, Ethereum and Ripple prices. The values are all statistically significant at the standard significance level and show that the degree of the long memory in the volatility process of the daily returns are quite different across crypto currencies. These estimation results are quite consistent with Figure 2 in the fact that the autocorrelations are apparently decaying very slowly at the hyperbolic rate in the squared and the absolute returns of the three crypto currencies. Thus, the long memory volatility can be considered as one of important inherent features in the daily returns of the crypto currencies. These findings are in line with the paper of Caporale *et al.* (2017) which find the evidence of the strong persistence in the crypto currencies.

The modified Ljung-Box test statistics are estimated by using the standardized residuals and presented by the values of the Q(20) and the $Q^2(20)$. The statistics show that the FIGARCH model applied for the daily returns

appears to be quite good in capturing the autocorrelations in the mean process and the variance process of the daily return series. Also, the two statistics provide evidence that there is no additional autocorrelation in the standardized residuals or squared standardized residuals. Some additional diagnostic portmanteau tests on the standardized residuals and squared standardized residuals also did not detect any need to further complicate the model so that the selected model specification seems to be the best fit. Thus, the FIGARCH model appears to represent the long memory volatility property quite well in the daily returns of the crypto currencies and the FIGARCH model under the Gaussian normality assumption generally seems to be appropriate in matching the dynamics of the daily returns of the crypto currencies. Thus, the FIGARCH model with the normality assumption can be considered as a good starting point to study the long memory feature in the crypto currency prices.

However, the estimated kurtosis from the FIGARCH model are found to be 7.75, 6.08 and 19.78 for the daily returns of Bitcoin, Ethereum and Ripple crypto currencies leading to the significant excess kurtosis in the daily returns of the crypto currencies even though the long memory volatility is accounted for by the FIGARCH model. The excess kurtosis may be related to the jumps in the daily returns of the crypto currencies, which are caused by order flow imbalance, illiquidity and the dominant effect of aggressive traders. Since these jumps can cause the "outliers" in the mean and the volatility process, the simple normal distribution model may not take account for the jumps appropriately (Hotta and Tsay, 1998). Thus, the simple FIGARCH model with the Gaussian normal distribution may not be appropriate to represent the jumps and the long memory volatility together in the daily returns of the crypto currencies.

⟨Table 2⟩ Estimated Martingale–FIGARCH(1,d,1) Model for the Daily Returns of Three Crypto Currencies

	Bitcoin	Ethereum	Ripple
μ	0.2288[*]	0.1344	−0.3395[*]
	(0.1219)	(0.1617)	(0.1766)
d	0.3763[*]	0.2691[***]	0.5411[***]
	(0.2144)	(0.0761)	(0.1097)
ω	0.0524	0.1546	0.2404
	(0.8445)	(0.1342)	(0.2453)
β	0.5558[***]	0.9821[***]	0.9769[***]
	(0.1521)	(0.0181)	(0.0222)
φ	0.3167[***]	0.9676[***]	0.9918[***]
	(0.1117)	(0.0300)	(0.0199)
ln(L)	−2487.970	−2783.627	−2872.296
m3	0.055	−0.013	1.391
m4	7.759	6.085	19.782
Q(20)	24.739	22.595	26.977
Q^2(20)	13.710	9.954	29.127

Notes: 1) QMLE asymptotic standard errors are in parentheses below corresponding parameter estimates. The quantity ln(L) is the value of the maximized log likelihood. m3 and m4 represent the sample skewness and kurtosis of the standardized residuals. Q(20) and Q^2(20) statistics are the Ljung-Box test statistics for 20 degrees of freedom to test for serial correlation in the standardized residuals and the squared standardized residuals.

   2) the asterisks(*,**,***) denote significance at 10%, 5% and 1% level respectively.

IV. Bernoulli Jumps and Long Memory Volatility in Daily Crypto Currency Prices

The simple FIGARCH model in Section III appears to be useful in describing the long memory volatility process but it seems to be inappropriate to explain the jumps as the cause of the long memory property. Recently, many studies have been more focus on explaining the underlying causes or sources for the long memory volatility process of financial assets. Some papers have provided possible sources of the long memory volatility process by using nonlinear volatility models including structural breaks (Granger and Hyung, 2004; Starcia

and Granger, 2004; Choi and Zivot, 2007; Kwon, 2010; Han, 2011) or jumps (Beine and Laurent, 2003; Han, 2007 and 2011). They have suggested that the observed long memory volatility process may be caused by the structural breaks or the jumps. Even though the jump is closely related to the structural breaks, the main differences between the jumps and the structural breaks are that the jumps in volatility create fat-tailness and high excess kurtosis in the price process and that the jumps are more drastic but short lived relative to the structural breaks.

While some studies have analysed the long memory property in crypto currency prices by adopting the form of switching regimes or structural breaks (Bariviera, 2017; Caporale *et al.,* 2018; Yonghong *et al.,* 2018; Mensi *et al.,* 2019), other papers have presented the importance of jumps in the price process of the crypto currencies. Gronwald (2015) empirically analyzes the Bitcoin prices by using autoregressive jump intensity GARCH model and finds strong evidence of jump behavior. And, Scaillet *et al.* (2017) recently finds that the jumps are frequent and they cluster in time in the price process of Bitcoin market indicating that the jumps are an essential component of the price process of the Bitcoin and they show that the jumps can be driven by order flow imbalance, illiquidity and the dominant effect of aggressive traders and they have short term positive impacts on market activity inducing a persistent change in the price process. Thus, analyzing the properties of the jumps could be important due to the consequences in applications of derivatives pricing and risk managements in the crypto currency market.

This section considers the specification of the jumps and the long memory volatility in price process of the daily crypto currencies.[4] For the purpose, this

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4) Andersen *et al.* (2002) specified the model of exchange rate dynamics by allowing for the possibility of jumps and demonstrated that the volatility estimation can be better by including jumps. Beine and Laurent (2003) and

paper adopts the jump diffusion process of Press (1967) to consider the jumps in the daily returns of the three crypto currencies. In particular, this paper uses a Bernoulli distribution to specify the stochastic jumps in the price process of the daily crypto currency returns series. Even though stochastic jumps have been modelled by using the Poisson distribution, the Bernoulli jump process can be practically much easier and more convenient in specifying the jumps caused by new information arrivals than the Poisson distribution because the Bernoulli process is much simpler in calculation without requiring the infinite sum and the truncation process required by the Poisson process. Thus, this study adopts the Bernoulli jump process to specify the jumps and combines it with FIGARCH model to analyze the impact of the jumps on the long memory volatility process of the daily crypto currency returns series.

The key feature of the Bernoulli process is that no information impacts on the price or one relevant information arrival within a fixed time period (t) happens with probability λ which is in the (0,1) interval and can be found by estimating the parameter (j) in the expression, $\lambda = [1 + \exp(j)]^{-1}$. The jump size is considered as the random variable (v), assuming to fellow NID (ν, δ^2). The same FIGARCH model as in Section III is used for the long memory volatility process. Because the jumps and the long memory property have quite different statistical and economic motivations, this paper selects a model specification which can consider the two features simultaneously.

This paper investigates the daily returns of the crypto currencies (Bitcoin, Ethereum and Ripple) by employing the Bernoulli jump diffusion process to consider the jumps and the excess kurtosis, combined with the FIGARCH

Han (2007, 2008) showed that the jumps might increase the volatility and generate strong long memory volatility of foreign exchange rates.

model to capture the long memory volatility. By including the jump process, the influence of the jumps on the FIGARCH specification can be reduced. The combined Martingale-FIGARCH (1, d, 1) model with Bernoulli jump process is,

$$y_t = \mu + \lambda \nu + \epsilon_t \tag{8}$$

$$\epsilon_t = z_t \sigma_t \tag{9}$$

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + [1 - \beta L - (1 - \phi L)(1 - L)^d] \epsilon_t^2 \tag{10}$$

The daily returns are still specified as the same Martingale process with the jump intensity of λ and the jump size of v. The volatility process is the same FIGARCH(1,d,1) as developed in Section III. The log likelihood function for the specified model is,

$$\ln(L) = -\left(\frac{T}{2}\right)\ln(2\pi) + \sum_{t=1}^{n} \ln\left[\left(\frac{(1-\lambda)}{h_t}\right)*\exp\left(-\frac{(\epsilon_t + \lambda v)^2}{2h_t^2}\right) + \tag{11}$$
$$\left(\frac{\lambda}{(h_t^2 + \delta^2)^{\frac{1}{2}}}\right)*\exp\left(-\frac{(\epsilon_t + (1-\lambda)v)^2}{2(h_t^2 + \delta^2)}\right)\right]$$

The likelihood function for the Normal-Bernoulli jump processes is basically the same as that presented in Vlaar and Palm (1993) and Baillie and Han (2001). Asymptotic standard errors are also based on the QMLE of Bollerslev and Wooldridge (1992) as in Section III.

The estimated parameters for the daily returns series of the three crypto currencies are reported in Table 3. The estimated values of the parameters (j) for the jumps in the daily returns are found to be 1.29, 3.53 and 1.73 and they are statistically significant. So, the values show that the Bernoulli jump process can be quite appropriate to specify the mean process of the daily returns. Also, the findings are quite consistent with the papers of Gronwald (2015) and Scaillet

et al. (2017) which present strong evidence of jumps behavior and assert that jumps are the essential component of the dynamics of the crypto currency prices. In particular, the jumps intensities (λ) calculated from the estimated parameters (j) are 0.21, 0.02 and 0.14 for Bitcoin, Ethereum and Ripple returns series respectively. By using the total 880 observations of Bitcoin, Ethereum and Ripple currency, the corresponding implied numbers of the jumps can be about 185, 18 and 123 for the daily returns series of the three crypto currencies. The Bitcoin appears to contain the most jumps in the price process since it has the largest trading volume and the most popular and prominent crypto currency with multi-billion dollar capitalization and the biggest market shares in the crypto currency market (Catarina and Grassi, 2017).

The estimated values of the parameter (ν) that indicate the impacts of the jumps on the mean process are all positive and statistically significant for the daily returns of the crypto currencies suggesting the jumps appear to increase the prices of the crypto currencies on average. Also the values of the parameter (δ^2) which represent the impacts of the jumps on the volatility process are also significant at the standard significance level and the values are found to be much greater than those on the mean process, suggesting that the effects on the volatility process are more significant.

In particular, the results in Table 3 show that the estimated long memory parameters (d) of the daily returns are 0.17, 0.08 and 0.05 for the daily returns of Bitcoin, Ethereum and Ripple respectively and are statistically significant except Ehtereum. It is interesting to note that the values of the estimated long memory parameters appear to be decreased significantly compared with the values estimated from the simple FIGARCH model without the jumps in Section III. This suggests that the great long memory volatility of the daily returns may be significantly affected by the jumps and the greater long

memory parameters can be distorted spuriously because the jumps are not accounted for properly in the price process of the crypto currencies. These results appear to be understandable when the jumps are entirely considered in the mixture distribution since the jumps can cause the additional volatility spuriously. Thus, the higher long memory volatility property seems to be related to the asymmetric price adjustments to the jumps, which is much more gradual and persistent than the volatility adjustments and the jumps can be considered as the possible driving forces of the long memory volatility process of the daily crypto currencies prices.

The estimation results in Table 3 show that the kurtosis statistics (m4) are also reduced significantly for the daily returns after the jumps are considered suggesting that accounting for the non-uniform flows of information can reduces the excess kurtosis. Thus, the results present that the mixture distribution generally seems to be better than the simple normal distribution and quite appropriate to explain the jumps in the price process of the crypto currencies.[5]

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5) The data of Bitcoin sampled from April 28, 2013 to May 31, 2019 is also tested for the analysis of the long memory volatility and the jumps, and the results are found to be quite similar. But, the trading activities and the price movements of the Bitcoin in the markets before 2017 are very small and can not provide any economic significance.

〈Table 3〉 Estimated Martingale-FIGARCH(1,d,1) Model with Bernoulli Jump
Process for the Daily Returns of Three Crypto Currencies

	Bitcoin	Ethereum	Ripple
μ	0.2803***	-0.1755	-0.3651***
	(0.1020)	(0.1538)	(0.1068)
j	1.2997***	3.5301***	1.7393***
	(0.3256)	(0.4266)	(0.2931)
ν	0.97814**	4.0443***	4.5810***
	(0.4950)	(0.7853)	(2.1002)
δ^2	42.3355***	9.7157***	178.4927***
	(8.0901)	(1.2202)	(64.9542)
d	0.1700***	0.0835	0.0547***
	(0.0537)	(0.0829)	(0.0197)
ω	1.9063***	1.3809	1.9054*
	(0.4089)	(1.8031)	(1.0332)
β	0.4881***	0.8043***	0.4836***
	(0.1303)	(0.1352)	(0.1168)
φ	0.3946***	0.8784***	0.6415***
	(0.1277)	(0.1028)	(0.1214)
ln(L)	-2391.824	-2767.711	-2748.489
m3	-0.254	0.264	0.586
m4	3.329	2.545	5.015
Q(20)	30.231	20.803	26.557
Q^2(20)	1.970	9.147	10.970

Notes: 1) the same as Table 2 except that a jump intensity of λ, where $0<\lambda<1$ and $\lambda = [1 + \exp(j)]^{-1}$, and is specified by the Bernoulli process. The jump size is given by the random variable v_t, which is assumed to be NID(ν, δ^2).
2) the asterisks(*,**,***) denote significance at 10%, 5% and 1% level respectively.

V. Conclusions

This paper considers the intrigue time series properties of three major crypto currencies, Bitcoin, Ethereum and Ripple focusing on the jumps and the long memory volatility process of the daily prices. The general results of this paper present i) that the normality assumption may be inappropriate for the daily returns data of the crypto currencies mainly due to the existence of the jumps in the price process, ii) that most of the jumps could be related to

order flow imbalance, illiquidity and the dominant effect of aggressive traders in the crypto currency market and iii) that the Martingale-FIGARCH(1,d,1) model with the Bernoulli jump process can be quite good in explaining the jumps and the long memory volatility process of the daily returns of the crypto currencies. In particular, the results present that the long memory volatility becomes significantly weaker when the jumps are accounted for so that it may be closely related to the jumps providing statistical evidence that the specification of the price process without considering the jumps seem to increase the volatility of the daily returns and may distort the estimates of the long memory volatility parameters.

This paper seems to be quite useful in increasing our understanding of the dynamic price behaviors of the crypto currencies and their long memory volatility property. In particular, the results suggest that such a representation model is a possible alternative modeling strategy to long memory model with a normal distribution or to account for the jumps in the dynamics of the crypto currency. Thus, this paper provides important implications in the aspects of risk management and investment purpose for choosing a reliable model forecasting the risk in such an investment as this market attracts increasing attraction from regulators and investors as the new crypto currency market is becoming an important piece of financial technology.

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요 약

본 논문은 일별 암호화폐 가격들에 내재된 장기기억변동성과 점프현상을 파악한다. 이러한 목적을 위해 본 논문은 먼저 단순한 FIGARCH 모형을 통해 일별 암호화폐 가격들의 수익률에서 나타나는 장기기억 변동성 특성을 분석하여 암호화폐의 수익률에서 지속적인 장기기억 변동성이 존재함을 밝혔다. 또한, 암호화폐의 수익률에서 명확한 평균 점프현상이 유이하게 나타남에 따라 일반적인 정규분포 가정이 적절하지 못함을 파악하였다. 따라서 본 논문에서는 일별 암호화폐의 수익률 과정에 Bernoulli 점프 현상을 결부한 정규 혼합 분포를 이용한다. 특히, 평균과정에서의 점프현상이 일별 암호화폐의 수익률 과정에서 장기기억 변동성을 보다 더 크게 야기하는 것으로 보인다. 이에 따라 점프현상은 암호화폐들에 중요한 영향을 미치며, 특히 가격들의 지속적인 변동성을 야기한 것으로 파악되었다. 이러한 결과들을 통해 점프현상을 고려하지 않은 단순한 FIGARCH 모형은 암호화폐들의 장기기억 변동성 과정을 적절히 나타내지 못하며, 이로 인해 암호화폐 시장에서 효율적인 위험관리나 포트폴리오의 최적화를 이룰 수 없게 되는 결과가 나타날 수 있다. 따라서 본 논문은 시장규제자들과 투자자들의 많은 관심을 끌고 있는 암호화폐 시장에서 위험관리와 투자의 측면에서 위험예측을 위해서는 점프현상을 고려한 FIGARCH모형이 보다 적합함을 보여준다.

※국문 색인어 : 일별 암호화폐 가격, 정규혼합 분포, 베르누이 점프현상, FIGARCH 모형, 장기기억변동성