Monopoly Pricing of Weather Index Insurance

Tim J. Boonen Wenyuan Li Zixiao Quan
The University of Hong Kong

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Outline

Motivation: Weather Risks & Index Insurance

- **Agricultural Vulnerability:** Weather-based risks severely threaten agricultural livelihoods and the socioeconomic stability of rural communities.
- **Traditional Indemnity Insurance:** Widely adopted but suffers from high administrative costs, moral hazard, and relies heavily on subjective loss assessments.
- **Index Insurance:**
  - Connects payoffs to objective, verifiable weather indices.
  - Mitigates moral hazard and lowers transaction costs.
- **The Core Challenge:** Designing contracts that materially hedge farmer risk (minimizing *basis risk*) while remaining profitable. Especially in presence of high-dimensional and nonlinear weather-yield relationships as for weather index insurance.

Our Contributions

- **Game-Theoretic Framework:** We model monopoly pricing as a Bowley-type sequential game (Insurer as leader, Farmer as follower).
- **Deep Learning Architectures:** We compare Fully Connected Neural Networks (NNs) with Convolutional Neural Networks (CNNs) for modeling index payoffs, which demonstrates how CNNs efficiently reduce basis risk.
- **Pricing Flexibility:** We analyze three premium strategies:
  1. Single risk-loading expected premium.
  2. Two-parameter distortion premium.
  3. Fully flexible pricing kernel.
- **Algorithmic Innovation:** We solve the game using a penalized bilevel programming algorithm with a function-value-gap penalty, which requires no strong convexity assumptions on the lower level.

Literature Review: Bowley Solutions in Insurance

- **Monopoly Pricing:** Typically modeled as a leader-follower game where the insurer sets premiums and the policyholder chooses optimal coverage (Chan and Gerber, 1985).
- **Research Gap:** The current Bowley literature focuses almost exclusively on *indemnity* insurance contracts written on actual incurred losses.
- **Our Extension:** We apply Bowley equilibrium concepts specifically to the design and pricing of *weather index* insurance in concentrated markets.

Literature Review: Index Insurance & Basis Risk

- **Nonlinear Relationships:** Weather-yield relationships are highly complex. Flexible models like B-splines (Tan and Zhang, 2024) and standard Neural Networks (Chen et al., 2024) have been proposed to align payoffs and reduce basis risk.
- **Methodological Gap:** While standard NNs improve upon linear models, they require flattening 2D weather-time matrices. This destroys the structural context of the weather.
- **Our Extension:** We introduce CNNs to directly process the index-time matrix as an "image", which captures hierarchical, localized weather patterns to further minimize basis risk (Krizhevsky et al., 2012; Chen et al., 2024).

Sequential Game Formulation

- **The Farmer (Follower):** Faces a stochastic production loss Y. Chooses insurance payoff function $I(\mathbf{X})$ to minimize a distortion risk measure ρ_F.
- **The Insurer (Leader):** Anticipates farmer's best response. Chooses premium principle parameters to maximize profit net of administrative costs μ.

Upper Problem (UP): Insurer

$$\max_{\Pi \in \mathcal{P}_s} \Pi(I(\mathbf{X})) - (1+\mu)\mathbb{E}(I(\mathbf{X}))$$

Lower Problem (LP): Farmer

$$\min_{I \in \mathcal{I}} \rho_F(Y - I(\mathbf{X}) + \Pi(I(\mathbf{X})))$$

$$\text{s.t.} \quad I(\mathbf{X}) \geq 0$$

Distortion Risk Measures

Both parties evaluate risks using distortion functions $g : [0, 1] \to \mathbb{R}_+$.

Farmer's Preferences (ρ_F):

- *Conditional Value-at-Risk (CVaR):* Focuses on tail risk. $g(s) = \min\{\frac{s}{1-\alpha}, 1\}$.
- *Convex Combination:* Balances CVaR and expected risk.

$$g_f(s) = \lambda s + (1 - \lambda) \min \left\{ \frac{s}{1 - \alpha}, 1 \right\}$$

Insurer's Premium Principles (Π):

- **Problem 1 ($\mathcal{P}_{s1}$):** Expected premium with loading θ.
- **Problem 2 ($\mathcal{P}_{s2}$):** Power distortion premium (parameters θ, ρ).
- **Problem 3 ($\mathcal{P}_s$):** Fully flexible pricing kernel g_i.

Data Overview

- **Yield Data:** Annual county-level soybean data for Iowa (NASS), covering 1940-2023. Normalized and detrended to extract pure weather-induced production losses.
- **Weather Data:** High-dimensional PRISM Climate Group data.
- 7 climatic indices $\times$ 12 months = 84-dimensional grid.

Variable	**Description**
pcpnk	Total precipitation (rain and melted snow)
tmaxk	Daily maximum temperature
tmink	Daily minimum temperature
dptk	Daily mean dew point temperature
vpdmaxk	Daily maximum vapor pressure deficit
vpdmink	Daily minimum vapor pressure deficit

Neural Network Models (NN vs. CNN)

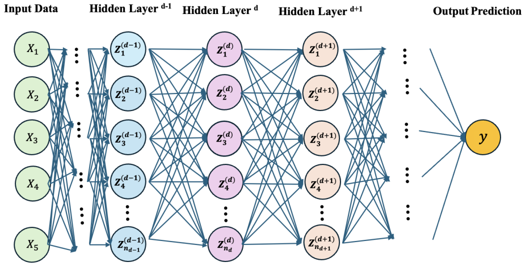


Figure 1: Traditional Fully Connected Neural Network (NN)

- **Limitation:** NNs require flattening the 2D weather-time matrix, which destroys the structural, spatial-temporal relationships of weather patterns.

Convolutional Neural Networks (CNNs)

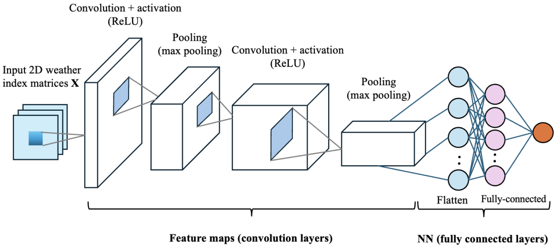


Figure 2: Convolutional Neural Network (CNN) Structure

- **Advantage:** Treats the index-time matrix as an "image."
- Uses convolutional kernels to extract local features hierarchically.
- Weight sharing makes the model robust to shifts, which reduces overfitting.

Bilevel Programming Challenges

The generic bilevel problem is:

$$\mathcal{BP} : \min_{x,y} f(x,y) \text{ s.t. } x \in \mathcal{C}, y \in \mathcal{S}(x) := \arg \min_{y \in \mathcal{U}(x)} h(x,y) \tag{1}$$

The Challenge:

- The upper problem relies precisely on the solution set $\mathcal{S}(x)$ of the lower problem.
- Because our lower-level objective $h(x,y)$ (farmer's CVaR) is a linear function of order statistics, it is polyhedral and convex, but **not strongly convex**.
- Traditional implicit gradient methods fail here.

Value-Gap Penalty Reformulation

We transform the nested problem into a single-level problem by introducing a penalty $p(x, y)$:

$$\mathcal{BP}_{\gamma p} : \min_{x,y} F_\gamma(x, y) = f(x, y) + \gamma p(x, y) \text{ s.t. } x \in \mathcal{C}, y \in \mathcal{U}(x) \qquad (2)$$

Function-Value-Gap Penalty:

$$p(x, y) = h(x, y) - v(x), \quad \text{where } v(x) := \min_{y' \in \mathcal{U}(x)} h(x, y')$$

- Because $h(x, \cdot)$ satisfies a *weak sharp minimum* on compact sets, this penalty acts as a valid squared-distance bound.
- By applying **Danskin's Theorem**, we can approximate the subgradient of the value function $\nabla v(x)$ using any optimal lower-level solution y^*.

Algorithm: PBGD Implementation

Algorithm 1 PBGD for Expected Premium

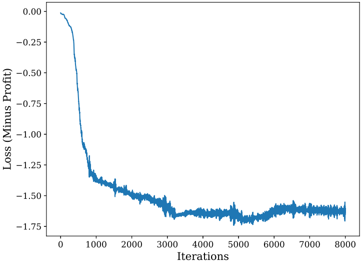
1: **Input:** Neural networks θ_{nn}, $\hat{I}^{\theta}_{nn}$, I^{θ}_{nn}; parameters $\alpha_0, \lambda, \gamma, \mu, N_L, N_C$.
2: **for each** iteration $n_C = 1, \dots, N_C$ **do**
3: **for each** LP objective iteration $n_L = 1, \dots, N_L$ **do**
4: Assign weights $I^{\theta}_{nn} \to \hat{I}^{\theta}_{nn}$; Compute step size $\alpha^{(L)}_{n_L}$;
5: Update $\hat{I}^{\theta}_{nn}$ by gradient descent on the LP objective.
6: **end for**
7: Compute $\mathrm{LP}(\hat{I}^{\theta}(\mathbf{X}))$ using $\hat{I}^{\theta}_{nn}$;
8: Compute penalty $\gamma \cdot \Big(\mathrm{LP}(I^{\theta}(\mathbf{X})) - \mathrm{LP}(\hat{I}^{\theta}(\mathbf{X}))\Big)$;
9: Compute combined objective:

$$-\Big(\Pi(I^{\theta}(\mathbf{X})) - (1+\mu)\mathbb{E}(I^{\theta}(\mathbf{X}))\Big) + \gamma \cdot \Big(\mathrm{LP}(I^{\theta}(\mathbf{X})) - \mathrm{LP}(\hat{I}^{\theta}(\mathbf{X}))\Big)$$

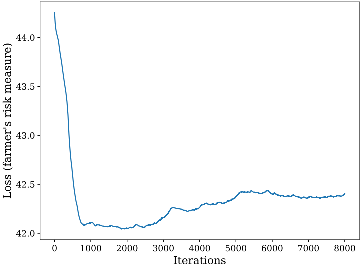
10: Update θ_{nn} and I^{θ}_{nn} by gradient descent on combined objective.
11: **end for**
12: **Output:** Optimal θ and $I(\mathbf{X})$.

Problem 1: Training Convergence (NN, Convex Combo)

Farmer's risk measure: $g_f(s) = 0.5s + 0.5\min\left\{\frac{s}{1-0.8}, 1\right\}$



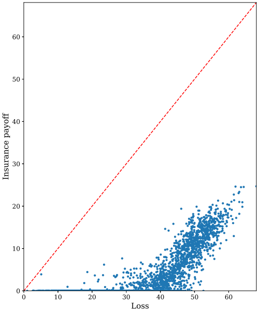
(a) Upper-problem loss (Negative Profit)



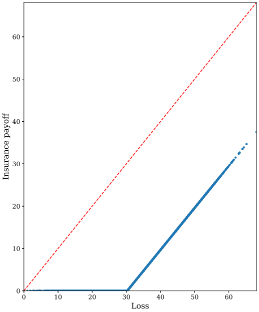
(b) Lower-problem loss (Farmer's Risk)

Key Insight: Loss curves stabilize indicates convergence to an equilibrium.

Problem 1: Payoff Comparison (Convex Combo $\lambda = 0.5$)



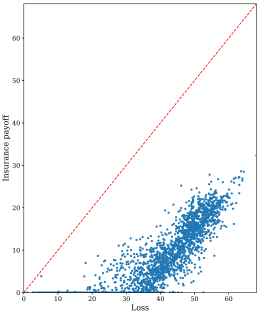
(a) Index Insurance



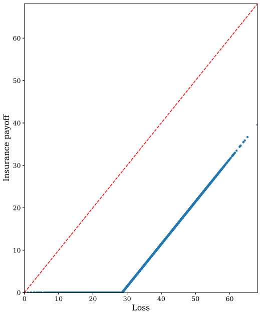
(b) Indemnity Insurance

Key Insight: Optimal policies exhibit stop-loss shapes. Index insurance ($\theta^* = 0.4130$) contains significant scatter noise compared to the pure Indemnity baseline ($\theta^* = 0.2812$).

Problem 1: Payoff Comparison (Pure CVaR)



(a) Index Insurance

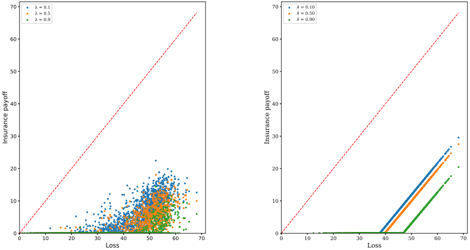


(b) Indemnity Insurance

Key Insight: When the farmer acts strictly on CVaR, the stop-loss shape persists, but the insurer applies a much higher risk loading for Index insurance ($\theta^* = 0.5500$) compared to Indemnity insurance ($\theta^* = 0.4853$).

Problem 1: Sensitivity Analysis (Varying λ)

Farmer's Risk Aversion varies: $\lambda = 0.1, 0.5, 0.9$



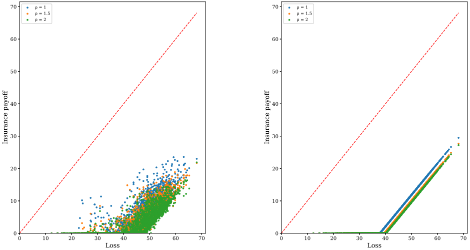
(a) Index Insurance (b) Indemnity Insurance

λ	**0.1**	**0.5**	**0.9**
Index θ^*	0.7542	0.5578	0.3068
Indemnity θ^*	0.6451	0.3724	0.2591

Key Insight: Higher λ (less risk-averse) leads to higher deductibles and lower optimal risk loading (θ^*).

Problem 1: Sensitivity Analysis (Varying ρ)

Insurer's Premium Distortion varies: $\rho = 1.0, 1.5, 2.0$



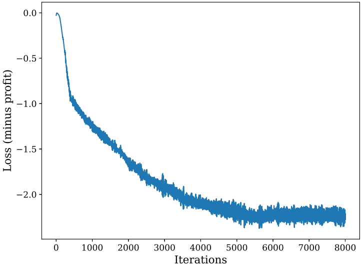
(a) Index Insurance

(b) Indemnity Insurance

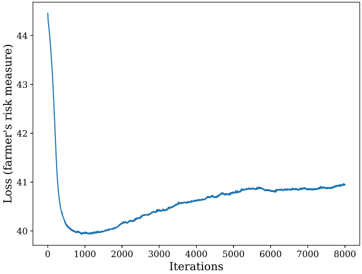
ρ	**1.0**	**1.5**	**2.0**
Index θ^*	1.0181	1.0333	1.0802
Indemnity θ^*	0.8796	0.9382	0.9855

Key Insight: A larger ρ (heavier tail loading by the insurer) slightly raises the deductibles for the farmer and increases the risk loading θ^*.

Problem 1: CNN Training Convergence



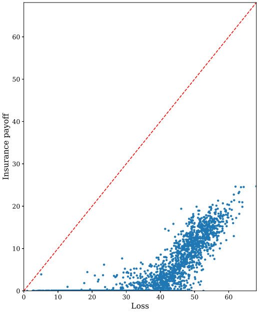
(a) UP Loss (Negative Profit)



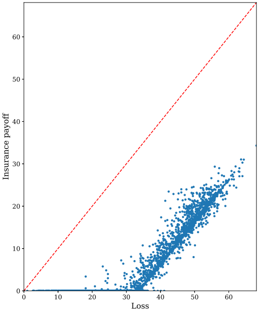
(b) LP Loss (Farmer's Risk)

Key Insight: Using CNNs to model the payoff yields a smoother and more robust training convergence pattern.

Problem 1: Payoff Modeling (Combo: NN vs. CNN)



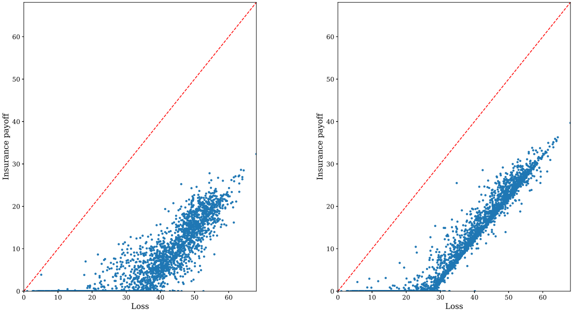
(a) NN: Index (Combo)



(b) CNN: Index (Combo)

- **Noise Reduction:** CNNs capture the spatial-temporal weather grids accurately, which visually removes the scattered basis risk seen in standard NNs.

Problem 1: Payoff Modeling (CVaR: NN vs. CNN)



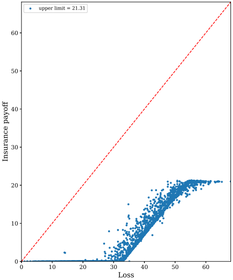
(a) NN: Index (Pure CVaR)

(b) CNN: Index (Pure CVaR)

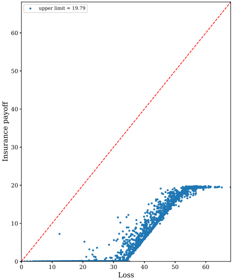
- **Profit Maximization:** By reducing the basis risk seen on the left, our CNN models pushed the index profit up to 4.07—significantly closer to the theoretical pure indemnity maximum of 4.82.

Problem 2: Two-Parameter Premium (Index)

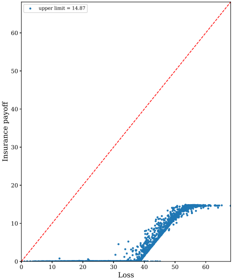
Insurer optimizes both θ and ρ. Farmer uses convex combination (λ).



(a) $\lambda = 0.1$ ($\theta^* = 0.2237, \rho^* = 2.0644$)



(b) $\lambda = 0.5$ ($\theta^* = 0.0758, \rho^* = 1.7341$)

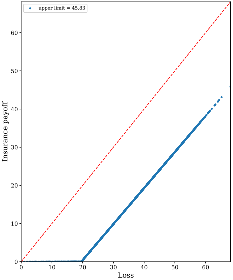


(c) $\lambda = 0.7$ ($\theta^* = 0.0190, \rho^* = 1.5016$)

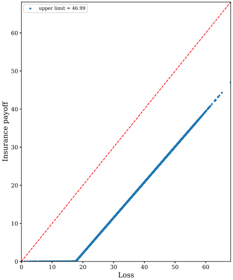
Key Insight: Extra pricing power creates a “limited stop-loss” shape (deductible + maximum cap). As the farmer becomes more risk-averse (lower λ), the insurer extracts more profit via higher θ and ρ.

Problem 2: Two-Parameter Premium (Indemnity)

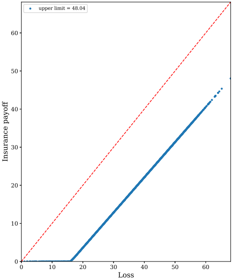
Theoretical baseline for the same parameters.



(a) $\lambda = 0.1$ ($\theta^* = 0.2761, \rho^* = 1.7078$)



(b) $\lambda = 0.5$ ($\theta^* = 0.1022, \rho^* = 1.5641$)

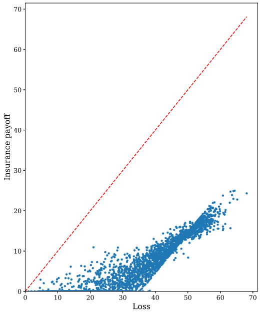


(c) $\lambda = 0.7$ ($\theta^* = 0.0258, \rho^* = 1.4565$)

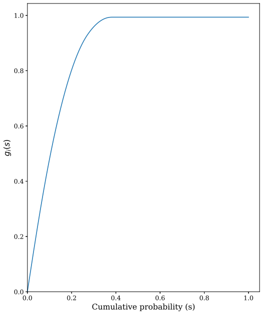
Key Insight: The indemnity counterpart maintains the clean, theoretical limited stop-loss shape. This allows the insurer to achieve slightly higher absolute profit ceilings.

Problem 3: Fully Flexible Pricing Kernel (Index)

The insurer uses a neural network to model $g_i(s)$ with total flexibility.



(a) Index Payoff

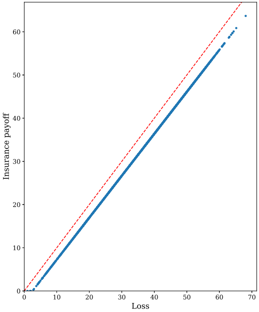


(b) Index Kernel $g_i(s)$

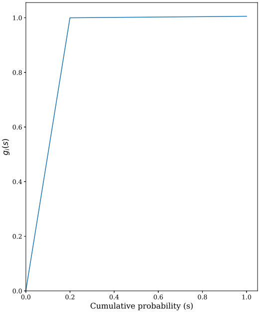
Key Insight: Given complete pricing flexibility, the optimal kernel is smooth and concave. The monopolistic insurer extracts an expected profit of **8.7961**—more than double the single-parameter model.

Problem 3: Fully Flexible Pricing Kernel (Indemnity)

Theoretical baseline under a flexible pricing kernel.



(a) Indemnity Payoff



(b) Indemnity Kernel $g_i(s)$

Key Insight: Interestingly, for pure indemnity, the optimal pricing kernel is piecewise linear with a kink at $s = 0.2$. This drives the deductible close to zero, which yields a theoretical profit of **17.2982**.

Conclusions

- **Architectural Impact:** Convolutional Neural Networks (CNNs) are highly effective at processing spatial-temporal weather grids. They yield significantly smoother, more robust payoff functions, systematically reducing basis risk compared to fully connected NNs.
- **Market Dynamics:** In concentrated markets, a monopolistic insurer effectively exploits farmer risk aversion. We demonstrated that granting the insurer greater pricing flexibility (from a scalar loading, to two parameters, to a full kernel) directly correlates with aggressive profit extraction.
- **Methodological Advance:** Our function-value-gap penalized bilevel programming algorithm scales successfully to large weather-yield datasets. It effectively solves sequential games without requiring strict strong convexity on the lower-level problem.

Thank you!

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